Regular Behaviours with Names

On the Rational Fixpoint of Endofunctors on Nominal Sets

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States & Transitions

(Functor-) Coalgebras

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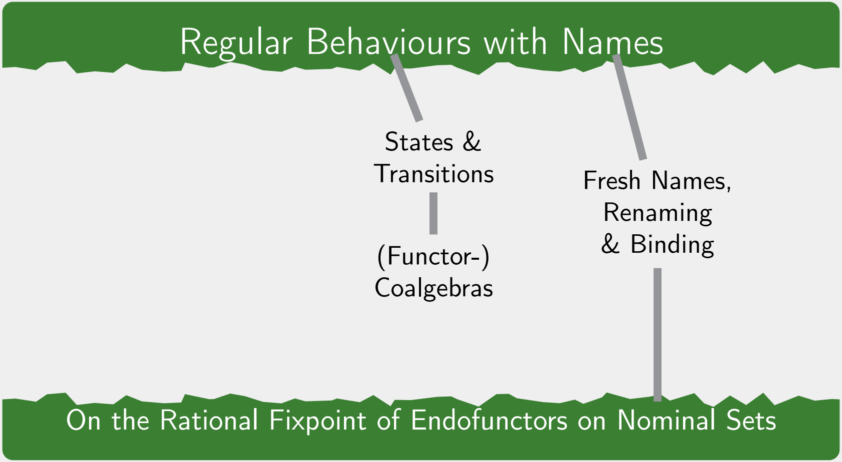
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Finite Description

Finitely Presentable Objects (orbit-finite in Nom)

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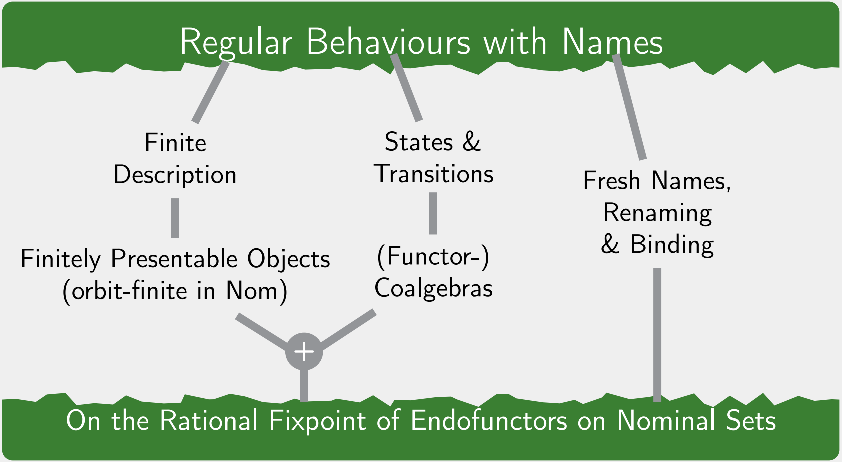
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The Framework of Nominal Sets

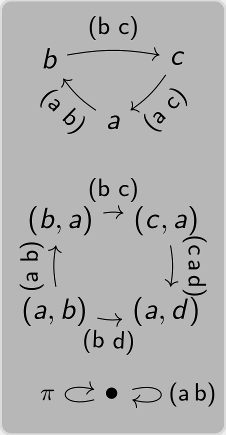
Finite permutations on $\mathcal{V}$

Support for a $\mathfrak{S}_f(\mathcal{V})$-action $\cdot : \mathfrak{S}_f(\mathcal{V}) \times X \to X$

"$S \subseteq \mathcal{V}$ supports $x \in X$", if for all $\pi \in \mathfrak{S}_f(\mathcal{V})$

$$\underbrace{\pi \text{ fixes } S}_{\pi(v)=v\ \forall v \in S} \implies \underbrace{\pi \text{ fixes } x}_{\pi \cdot x = x}$$

$\mathcal{V}^2 + 1 \cong$



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$(X, \cdot)$ a Nominal Set

"$\cdot$" a $\mathfrak{S}_f(\mathcal{V})$-action & every $x \in X$ finitely supported

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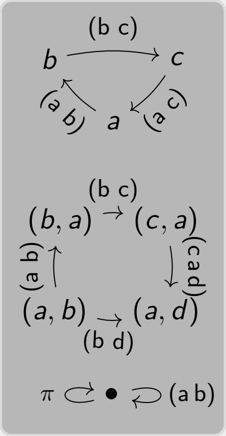

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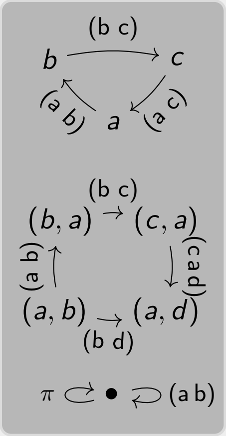

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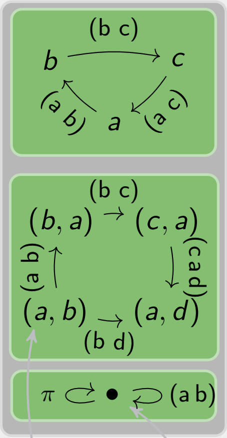

Either infinite

or singleton

What is it good for?

Instances of regular behaviours with names:

- Regular λ-trees

$$LX = \mathcal{V} + \mathcal{V} \times X + X \times X$$

- Regular λ-trees modulo α-equivalence

$$L_\alpha X = \mathcal{V} + [\mathcal{V}]X + X \times X$$

- Regular Nominal Automata

$$KX = 2 \times X^{\mathcal{V}} \times [\mathcal{V}]X$$

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- Regular λ-trees

Lifting of a Set-functor (Part 1)

$$LX = \mathcal{V} + \mathcal{V} \times X + X \times X$$

$$\downarrow q_X$$

- Regular λ-trees modulo α-equivalence

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Quotient of a lifting (Part 2)

- Regular Nominal Automata

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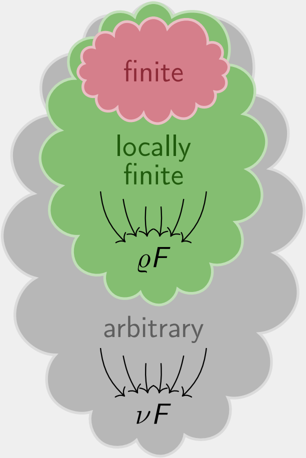
Part 1: Localizable Liftings

Regular Behaviours in Set

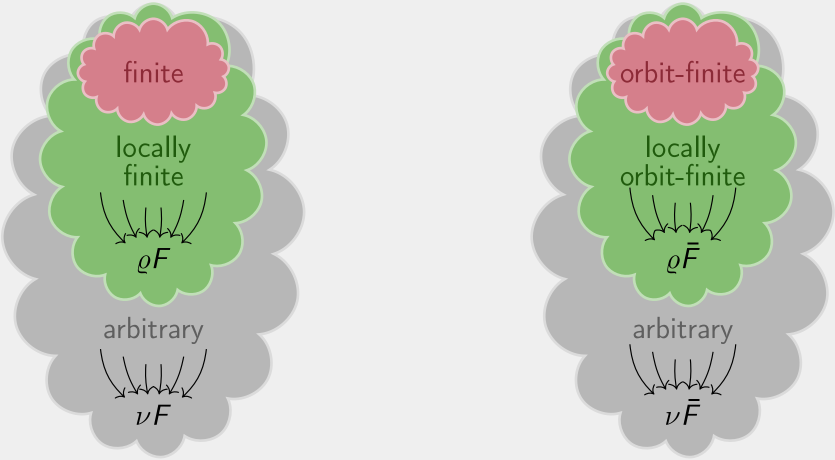
$F : \mathsf{Set} \to \mathsf{Set}$



Adámek, Milius, Velebil'06; Milius'10

Regular Behaviours in Set ... and in Nom

$F : \mathsf{Set} \to \mathsf{Set}$ $\bar{F} : \mathsf{Nom} \to \mathsf{Nom}$



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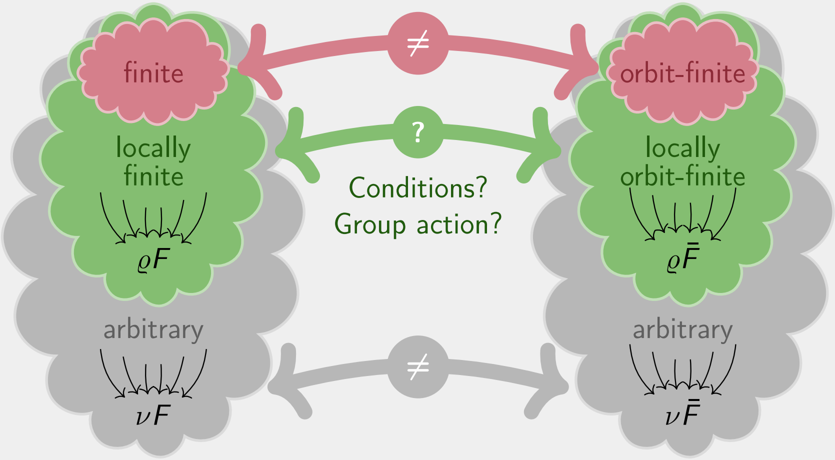
Regular Behaviours in Set ... and in Nom

If F : Set $\to$ Set ... lifts to ... $\bar{F}$: Nom $\to$ Nom



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Liftings

$\mathfrak{S}_{\mathsf{f}}(\mathcal{V})$-action on X

T-algebra structure on X for the monad $T = \mathfrak{S}_{\mathsf{f}}(\mathcal{V}) \times \_$

Liftings $\Longleftrightarrow$ Distributive Laws

$$\begin{array}{ccc} \mathsf{Set}^T & \xrightarrow{\bar{F}} & \mathsf{Set}^T \\ U \downarrow & & \downarrow U \\ \mathsf{Set} & \xrightarrow{F} & \mathsf{Set} \end{array} \quad \Longleftrightarrow \quad \begin{array}{c} \lambda : TF \to FT \\ \text{preserving} \\ \text{monad structure} \end{array}$$

Properties of liftings of $\mathfrak{S}_f(\mathcal{V}) \times \_$ over $F : \mathsf{Set}^{\circlearrowleft}$ (1)

$\bar{F}$ Nom-restricting

$\bar{F}$ maps nominal sets to nominal sets.

Examples

- Closed under finite products, coproducts, composition.
- For $(Y, \cdot)$ non-nominal, $KX = Y$ not Nom-restricting.

Properties of liftings of $\mathfrak{S}_f(\mathcal{V}) \times \_$ over $F : \mathsf{Set}^{\circlearrowleft}$ (2)

$\lambda : \mathfrak{S}_f(\mathcal{V}) \times F\_ \to F(\mathfrak{S}_f(\mathcal{V}) \times \_)$ localizable

For each $W \subseteq \mathcal{V}$, λ restricts to $\lambda : \mathfrak{S}_f(W) \times F\_ \to F(\mathfrak{S}_f(W) \times \_)$

Examples

- Closed under finite products, coproducts, composition, constants.
- For $F = \mathsf{Id}_{\mathsf{Set}}$, $\lambda(\pi, x) = (g \cdot \pi \cdot g^{-1})$ not localizable.

$\cong \mathsf{Id}_{\mathsf{Set}^T}$

Assumptions

Assumption: $\bar{F} : \mathsf{Nom} \circlearrowleft$ a localizable lifting, i.e.

1. $\bar{F}$ comes from a Nom-restricting distributive law λ over $F = U\bar{F}D$.
2. This λ is localizable

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Examples

- Constants, Identity.
- Closed under finite products, coproducts, composition.
- In particular: Polynomials in Nom
- $LX = \mathcal{V} + \mathcal{V} \times X + X \times X$
- For the strength of any finitary $F : \mathsf{Set} \circlearrowleft$ canonically defines a localizable lifting to Nom

LFP in Set vs LFP in Nom

Lemma

If for $c : C \to \bar{F}C$, the underlying $c : C \to FC$ is lfp in Set, then $c : C \to \bar{F}C$ is lfp in Nom.

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Corollary

$c : C \to \bar{F}C$ lfp in Nom iff the underlying $c : C \to FC$ is lfp in Set.

$(\varrho F, r)$ from Set to $\mathfrak{S}_{\mathsf{f}}(\mathcal{V})$-sets

Lemma

$(\varrho F, r)$ carries a canonical group action making r equivariant.

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Proof.

$$\mathfrak{S}_\mathsf{f}(\mathcal{V}) \times \varrho F \xrightarrow{\mathrm{id}\times r} \mathfrak{S}_\mathsf{f}(\mathcal{V}) \times F(\varrho F) \xrightarrow{\lambda_{\varrho F}} F(\mathfrak{S}_\mathsf{f}(\mathcal{V}) \times \varrho F)$$

is lfp because λ is localizable.
νF has canonical $\mathfrak{S}_\mathsf{f}(\mathcal{V})$-set structure (Bartels'04; Plotkin, Turi'97)
This map is just the restriction to ϱF. □

Coinduction

Definition: Coalgebra iteration

For $c : C \to HC$ put $c^{(n+1)} \equiv \left(C \xrightarrow{c^{(n)}} H^n C \xrightarrow{H^n c} H^{n+1} C \right)$.

Lemma

Let $H : \mathsf{Set} \circlearrowleft$ be finitary. If for H-coalgebras (C, c) and (D, d)

$$\begin{array}{ccccccc} X & \xrightarrow{p_1} & C & \xrightarrow{c^{(n)}} & H^n C & & \\ & \searrow^{p_2} & & & & \searrow^{H^n !} & \\ & & D & \xrightarrow{d^{(n)}} & H^n D & \xrightarrow{H^n !} & H^n 1 \end{array}$$

commutes for all $n < \omega$, then $c^\dagger \cdot p_1 = d^\dagger \cdot p_2$.

Finite support for ϱF

Lemma

Any $t \in \varrho F$ is supported by

$$s(t) = \bigcup_{n \geq 0} \mathrm{supp}(r^{(n)}(t)) \quad \text{where } r^{(n)} : \varrho F \to F^n(\varrho F)$$

and where the support of $r^{(n)}(t)$ is taken in $\bar{F}^n D(\varrho F)$.

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Lemma

For any $t \in \varrho F$, $s(t)$ is finite.

Universal Property

Theorem

The lifted $(\varrho F, r)$ is the rational fixpoint of $\bar{F}$.

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Proof.

Consider $c : C \to \bar{F}C$ with C orbit-finite.

1. c is lfp in Set, then $c^\dagger : (C, c) \to (\varrho F, r)$ in Set
2. Equivariant $j : (\varrho F, r) \rightarrowtail (\nu F, \tau)$ in $\mathfrak{S}_\mathsf{f}(\mathcal{V})$-sets ← Not in Nom
3. Equivariant $j \cdot c^\dagger : (C, c) \to (\nu F, \tau)$ in $\mathfrak{S}_\mathsf{f}(\mathcal{V})$-sets
4. $c^\dagger : (C, c) \to (\varrho F, r)$ equivariant

□

Examples

λ-trees $LX = \mathcal{V} + \mathcal{V} \times X + X \times X$

- $\varrho\bar{L}$ in Nom = rational λ-trees (not modulo α-equivalence)
- νL in Set = all λ-trees
- $\nu\bar{L}$ in Nom = λ-trees involving finitely many variables

Kurz, Petrisan, Severi, de Vries'13

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Canonical Liftings $\bar{F} : \text{Nom} \circlearrowleft$ of $F : \text{Set} \circlearrowleft$

$\varrho\bar{F}$ in Nom = ϱF with discrete nominal structure

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Canonical Liftings $\bar{F}$: Nom $\circlearrowleft$ of F : Set $\circlearrowleft$

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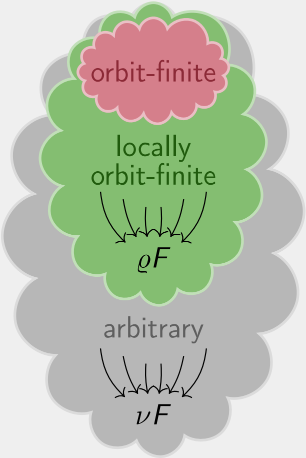
Unordered Trees: $FX = \mathcal{B}(X) + \mathcal{V}$

- νF = unordered trees with some leaves labelled in $\mathcal{V}$
- ϱF = those with finitely many subtrees
- $\varrho\bar{F}$ = those with renaming of the leaves

Part 2: Quotients of Nom-functors

Regular Behaviours in Nom

$F : \mathsf{Nom} \to \mathsf{Nom}$



Regular Behaviours in Nom … and their quotients

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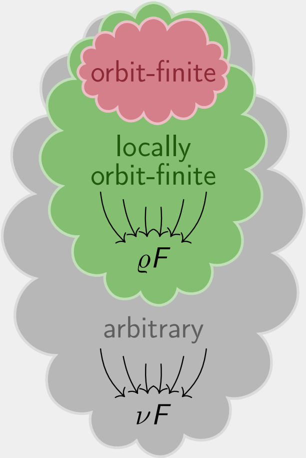
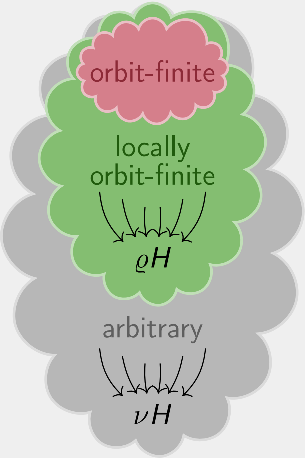


$H : \mathsf{Nom} \to \mathsf{Nom}$



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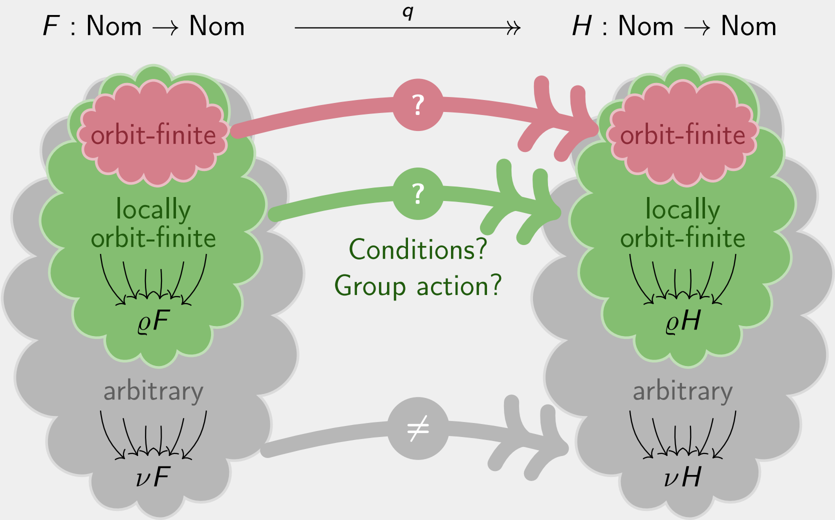

Quotients of coalgebras

$$F : \mathsf{Nom} \circlearrowleft \overset{q}{\twoheadrightarrow} H : \mathsf{Nom} \circlearrowleft$$

Definition: Quotient

A quotient from $a : A \to FA$ to $c : C \to HC$:

$$\text{some } h : A \twoheadrightarrow C \quad \text{with} \quad \begin{array}{ccccc} A & \xrightarrow{a} & FA & \xrightarrow{q_A} & HA \\ h\downarrow & & & & \downarrow Hh \\ C & & \xrightarrow{c} & & HC \end{array}$$

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If every orbit-finite H-coalgebra is a quotient of an orbit-finite F-coalgebra, then ϱH is a quotient of ϱF.

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Proof.

Epi-laws for jointly-epic the families. □

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How to prove that?

If every orbit-finite H-coalgebra is a quotient of an orbit-finite F-coalgebra, then ϱH is a quotient of ϱF.

Proof.

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Constructing a quotient backwards

Definition

$$X < Y = \{(x, y) \in X \times Y \mid \mathrm{supp}(x) \subseteq \mathrm{supp}(y)\}$$

Substrength of a functor F: $s_{X,Y} : FX < Y \to F(X < Y)$,

with $F\,\mathrm{outl} \cdot s_{X,Y} = \mathrm{outl}$ (not necessarily natural).

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Construction for $c : C \to HC$

$$B = \max_{x \in C} |\mathrm{supp}(x)| + \max_{x \in C} \min_{\substack{y \in FC \\ q_C(y) = c(x)}} |\mathrm{supp}(y)|.$$

$W \subseteq \mathcal{V}^B$ of tuples with distinct components.
F-Coalgebra on $C < W$.

Something like "projective objects" in Nom

Definition: strongly supported

Some $x \in X$ is **strongly supported** iff

$$\pi \cdot x = x \Longrightarrow \forall v \in \mathrm{supp}(x) : \pi(v) = v$$

Examples

W is strongly supported. $\mathcal{P}_\mathrm{f}(\mathcal{V})$ not.

Proposition (Mentioned already in Kurz, Petrisan, Velebil'10)

X, Y nominal sets, X strongly supported, $O \subseteq X$ a choice of one element from each orbit. Then any map $f_0 : O \to Y$ with

$$\mathrm{supp}(f_0(x)) \subseteq \mathrm{supp}(x)$$

extends uniquely to an equivariant $f : X \to Y$.

Applied to our $C < W$

Lemma

There is an equivariant map $f : C < W \to FC$ such that:

$$\begin{CD} C < W @>{f}>> FC \\ @V{\mathrm{outl}}VV @VV{q_C}V \\ C @>{c}>> HC \end{CD}$$

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Proposition

$c : C \to HC$ is via outl a quotient of the orbit-finite

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Corollary

If a finitary $F : \mathsf{Nom} \circlearrowleft$ has a substrength, and $q : F \twoheadrightarrow H$, then $\varrho F \twoheadrightarrow \varrho H$ (applying q level-wise).

Applicability

The only restricting requirement: F having a sub-strength

H and q: arbitrary

Lemma

1. Identity and constant functors have a sub-strength.
2. The class of functors with a sub-strength is closed under finite products, arbitrary coproducts, and functor composition.

Example: λ-trees modulo α-equivalence

$$LX = \mathcal{V} + \mathcal{V} \times X + X \times X \xrightarrow{q} L_\alpha X = \mathcal{V} + [\mathcal{V}]X + X \times X$$

Definition: Rational α-equivalence class of λ-trees

= contains some rational λ-tree

λ-trees

- ϱL = rational λ-trees
- νL = λ-trees with finitely many variables involved

λ-trees modulo α-equivalence

- ϱL_α = rational λ-trees modulo α-equivalence
- νL_α = λ-trees with finitely many **free** variables but possibly **infinitely** many bound variables

Kurz, Petrisan, Severi, de Vries'13

Example: Exponentiation

$$FX = \mathcal{V} \times X \times \coprod_{n\in\mathbb{N}} (\mathcal{V} \times X)^n \xrightarrow{q} (\_)^{\mathcal{V}}$$

Definition

$$\bar{q}_X(a, d, (v_1, x_1), \ldots, (v_n, x_n), b) = \begin{cases} x_i & \text{if } i = \min_{1\le j\le n}(v_j = b) \text{ exists} \\ (a\ b)\cdot d & \text{otherwise.} \end{cases}$$

Theorem: q component-wise surjective

For some $f \in X^{\mathcal{V}}$, $\{a_1, \ldots, a_n\} = \operatorname{supp}(f)$ and $a \in \mathcal{V} \setminus \operatorname{supp}(f)$, we have

$$\bar{q}_X(a, f(a), (a_1, f(a_1)), \ldots, (a_n, f(a_n)), b) = f(b) \text{ for all } b \in \mathcal{V}.$$

Example: Automata

Various Kinds of Nominal Automata

- $FX = 2 \times X^{\mathcal{V}}$
- $KX = 2 \times X^{\mathcal{V}} \times [\mathcal{V}]X$
- $NX = 2 \times \mathcal{P}_{\mathsf{f}}(X^{\mathcal{V}}) \times \mathcal{P}_{\mathsf{f}}([\mathcal{V}]X)$

Main Results

Final Coalgebra: Infinite Behaviours ⟷ Nominal Sets: finitely supported $\mathfrak{S}_f(\mathcal{V})$-action ⟷ Rational Fixpoint: orbit-finite Behaviours

Main Results

Final Coalgebra: Infinite Behaviours ⟵ ⚡ ⟶ Nominal Sets: finitely supported $\mathfrak{S}_\mathsf{f}(\mathcal{V})$-action ⟵ ♥ ⟶ Rational Fixpoint: orbit-finite Behaviours

⚡ fails

- If $\bar{F} : \mathsf{Nom} \circlearrowleft$ is localizable lifting of $F : \mathsf{Set} \circlearrowleft$ then $\varrho\bar{F}$ is ϱF with canonical $\mathfrak{S}_\mathsf{f}(\mathcal{V})$-action
- If $G : \mathsf{Nom} \circlearrowleft$ is a quotient $H : \mathsf{Nom} \circlearrowleft$ with a substrength then ϱG is a quotient of ϱH

Open Questions

About Localizable Liftings

- Is every non-localizable Lifting isomorphic to localizable one?
- If not, are there applications of non-localizable liftings?

About Substrengths

- Rational Fixpoint of quotients of functors without substrength?
- Are there applications?

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