

# Zensuren-Klassenspiegel

Nr.	Anzahl der Arbeiten	sehr gut = 1	gut = 2	befriedigend = 3	ausreichend = 4	mangelhaft = 5	ungenügend = 6	Klassen-durchschnitt	eigene Zensur	Unterschrift der Eltern
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## Übung - Aufgabe 1 PA1

$$a) \text{ Median } \bar{x} = (14,63 + 14,72) / 2 = 14,675$$

$$\text{unteres Quartil} = (u_{\lfloor \frac{25}{100} \cdot 20 \rfloor} + u_{\lceil \frac{25}{100} \cdot 20 \rceil}) / 2 = (14,05 + 14,23) / 2 = 14,14$$

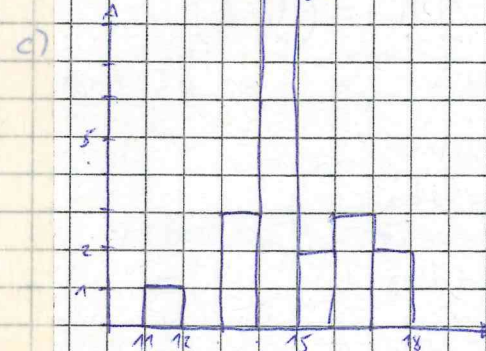
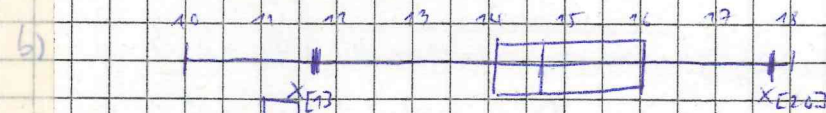
$$\text{oberes Quartil} = (u_{\lfloor \frac{75}{100} \cdot 20 \rfloor} + u_{\lceil \frac{75}{100} \cdot 20 \rceil}) / 2 = (15,81 + 16,27) / 2 = 16,04$$

$$16\% \text{ Quantil} = u_{\lfloor \frac{16}{100} \cdot 20 \rfloor} = u_{\lfloor 3,2 \rfloor} = u_3 = 13,70$$

$$84\% \text{ Quantil} = u_{\lfloor \frac{84}{100} \cdot 20 \rfloor} = u_{\lfloor 16,8 \rfloor} = u_{16} = 16,31$$

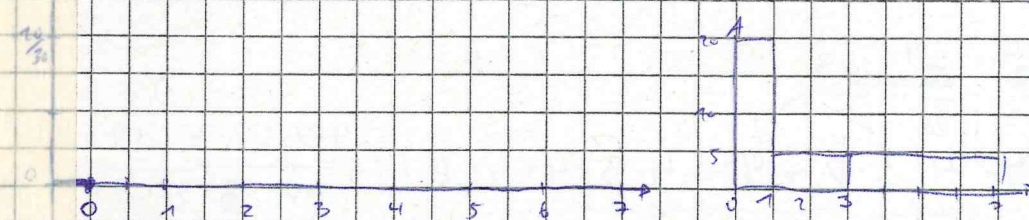
$$\text{arith. Mittel} = \frac{1}{20} \sum_{i=1}^{20} x_i = 14,9265$$

$$\text{Streuung} = \sqrt{\frac{1}{20} \sum_{i=1}^{20} (x_i - \bar{x})^2} = 1,395225$$



## Übung - Aufgabe 2 PA2

	abs.	rel. addiert
0	14	14/30
1	6	20/30
2	4	24/30
3	1	25/30
4	2	27/30
5	2	29/30
6	0	29/30
7	1	30/30



$$\begin{aligned} \text{PA3 } \left( \frac{1}{n} \sum_{i=0}^n (x_i - \bar{x})^2 \right)^{\frac{1}{2}} &= \left( \frac{1}{n} \sum_{i=0}^n x_i^2 - 2x_i \bar{x} + \bar{x}^2 \right)^{\frac{1}{2}} \\ &= \left( \frac{1}{n} \left( \sum_{i=0}^n x_i^2 + \sum_{i=0}^n \bar{x}^2 - 2x_i \bar{x} \right) \right)^{\frac{1}{2}} = \left( \frac{1}{n} \left( \sum_{i=0}^n x_i^2 + \sum_{i=0}^n \bar{x} (\bar{x} - 2x_i) \right) \right)^{\frac{1}{2}} \\ &= \left( \frac{1}{n} \left( \sum_{i=0}^n x_i^2 + \sum_{i=0}^n x_i \left( \frac{1}{n} \sum_{j=0}^n x_j - 2x_i \right) \right) \right)^{\frac{1}{2}} \\ &= \left( \frac{1}{n} \left( \sum_{i=0}^n x_i^2 - 2\bar{x} \frac{1}{n} \sum_{i=0}^n x_i + \frac{1}{n} \sum_{i=0}^n \bar{x}^2 \right) \right)^{\frac{1}{2}} = \left( \frac{1}{n} \left( \sum_{i=0}^n x_i^2 - 2\bar{x} \bar{x} + \bar{x} \bar{x} \right) \right)^{\frac{1}{2}} = \left( \frac{1}{n} \sum_{i=0}^n x_i^2 - \bar{x}^2 \right)^{\frac{1}{2}} \end{aligned}$$



Übung 2 - Aufgabe 8

23.04.18

- a)  $\Omega = \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\} = \{(a, b, c) \mid a, b, c \in \{1, \dots, 6\}\}$
- b)  $A = \{(x, y, z) \mid x, y, z \in \{1, 2, 3, 4, 5, 6\}, x=y=z\}$   
 $B = \{(x, y, z) \mid x, y, z \in \{1, 2, 3, 4, 5, 6\}, x+y+z \leq 3\}$   
 $C = \{(x, y, z) \mid x, y, z \in \{1, 2, 3, 4, 5, 6\}, \text{für } i, j: a_i = a_j = 6\}$

Aufgabe 9

- a)  $\Omega_n = \{1, 2, 3, 4, 5, 6\}^n$
- $A_k = \{x \in \Omega_n \mid x_k = 3\}$   
 $B_k = \{x \in \Omega_n \mid x_k = 3 \wedge \forall i (i < k: x_i \neq 3)\}$   
 $C = \{x \in \Omega_n \mid \exists k: x_k = 3 \wedge \forall i (i \neq k: x_i \neq 3)\}$   
 $D = \{x \in \Omega_n \mid \forall k: x_k \neq 3\}$
- $B_k = (\bigcap_{i=1}^{k-1} A_i^c) \cap A_k$   $C = \bigcup_{k=1}^n (\bigcap_{i=1, i \neq k}^n A_i^c) \cap A_k$   $D = \bigcap_{i=1}^n A_i^c = (\bigcup_{i=1}^n A_i)^c$

Aufgabe 10

- a) ja
- b) Fall:  $|\Omega| = 2$  O.B.d.A.  $\Omega = \{w_1, w_2\}, w_1 \neq w_2$   
 $\mathcal{A} = \{\emptyset, \{w_1\}, \{w_2\}, \{w_1, w_2\}\}$  ist eine  $\sigma$ -Algebra  
(i)  $\forall$  (ii)  $\forall$  (iii)  $\emptyset \cup \{w_1\}, \emptyset \cup \{w_2\}, \emptyset \cup \{w_1, w_2\} \in \mathcal{A}$  nachrechnen ✓  
Fall:  $|\Omega| \geq 3$  Betr.  $\mathcal{A}$  aus Angabe ist keine  $\sigma$ -Algebra  
Betr.  $w_1, w_2 \in \Omega, w_1 \neq w_2$   
dann ist  $\{w_1\} \cup \{w_2\} \notin \mathcal{A} \Rightarrow \mathcal{A}$  kein  $\sigma$ -Algebra

Aufgabe 15

- a)  $P(A \cap B) = P(A) - P(A \cap B^c) = P(A) - P(A \cap B^c) \leq 0 \checkmark$
- b) falsch:  $P(A) = \{w \mid w \text{ Anzahl Augen, } 1 \leq w \leq 3\}$   
 $P(B) = \{w \mid w \text{ " " " } 1 \leq w \leq 2\}$   $\Omega = \{1, 2, 3, 4, 5, 6\}$
- c) Ggbsp:  $A = \{w \mid w \in \{1, 2, 3\}\}$   $A^c = \{w \mid w \in \{3, 4, 5, 6\}\}$   
 $B = \{w \mid w \in \{1, 2, 3, 4, 5\}\}$   $B^c = \{w \mid w \in \{5, 6\}\}$

Aufgabe 16

- a)  $\frac{1}{40} \cdot \frac{1}{30} \cdot \frac{1}{38} \cdot \frac{1}{37}$
- b)  $\frac{1}{40 \cdot 30 \cdot 38 \cdot 37} \cdot (4 \cdot 4 \cdot 3 \cdot 2 + 4 \cdot 2 + 4 \cdot 3 \cdot 4 \cdot 3 \cdot 6) = \frac{4 \cdot 4 \cdot 3 \cdot 2 \cdot 4 \cdot 2 + 4 \cdot 3 \cdot 4 \cdot 3 \cdot 6}{40 \cdot 30 \cdot 38 \cdot 37}$
- c)  $\frac{10 \cdot 9 \cdot 8 \cdot 7}{40 \cdot 30 \cdot 38 \cdot 37} \cdot (40 \cdot 36 \cdot 32 \cdot 28)$

$\Omega$  besteht aus 40 unterscheidbaren Kugeln (Ziffer 0-9 und vier Farben), die unter Berücksichtigung der Reihenfolge gezogen werden

Aufgabe 17

$P(K) = 0,01$   $P(M|K) = 0,8$   $P(M|K^c) = 0,006$   $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(K | (M|K^c)) = \frac{P(K \cap M|K^c)}{P(M|K^c)} = \frac{P(K) - P(K \cap (M|K^c))}{P(M|K^c)}$

A-pos. Ergebnis  
B- Vorliegen von Brustkrebs

$P(A|B) = 0,8$   
 $P(A|B^c) = 0,006$   
 $P(B) = 0,01$   
 $P(B^c) = 0,99$

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A|B) \cdot P(B)}{P(A)}$

$= \frac{P(A|B) \cdot P(B)}{P(B^c) \cdot P(A|B^c) + P(B) \cdot P(A|B)} = \text{einsetzen} = 0,076$

Aufgabe 23

a)  $\sum_{n=1}^{\infty} f(x) = \sum_{n=1}^{\infty} C \cdot 3^{-n} = C \sum_{n=1}^{\infty} 3^{-n} = C \left( \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n - 1 \right) = C \cdot \left(\frac{1}{1-\frac{1}{3}} - 1\right)$

b)  $\int_{-\infty}^{\infty} C \cdot \exp(-\pi x - e^{-\pi x}) dx = C \cdot \int_{-\infty}^{\infty} e^{-\pi x} \cdot e^{-e^{-\pi x}} dx$

$= C \cdot \int_{-\infty}^{\infty} e^{-\pi x} \cdot e^{-e^{-\pi x}} dx$   $u = e^{-\pi x} \Rightarrow du = -\pi e^{-\pi x} dx$   
 $= C \cdot \int_{\infty}^{-\infty} (-1/\pi) e^{-u} du = \frac{C}{\pi} [e^{-u}]_{\infty}^{-\infty} = \frac{C}{\pi} [1 - 0] = \frac{C}{\pi}$   
 $\Rightarrow C = \pi$

Aufgabe 24

a)  $F(x) = \begin{cases} 0 & x \leq 1 \\ 1 - \frac{1}{x^2} & x > 1 \end{cases}$

b)  $F(x) = \begin{cases} 0 & x < \frac{1}{3} \\ x & \frac{1}{3} \leq x \leq \frac{1}{2} \\ 1 & x > \frac{1}{2} \end{cases}$

$\lim_{x \rightarrow 1/2^+} F(x) = \lim_{x \rightarrow 1/2^+} 1 = 1 \neq F(\frac{1}{2}) = \frac{1}{2}$

VF:  $F: \mathbb{R} \rightarrow [0, 1]$   
(i)  $F$  ist isotan  
(ii)  $\lim_{x \rightarrow -\infty} F(x) = 0, \lim_{x \rightarrow +\infty} F(x) = 1$   
(iii)  $\lim_{x \rightarrow x_0} F(x) = F(x_0)$

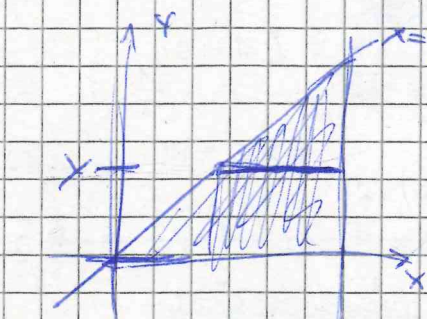


# Aufgabe 25

$$\begin{aligned} \text{a) } P(j < X \leq j+1) &= \int_j^{j+1} \lambda e^{-\lambda x} dx = -e^{-\lambda x} \Big|_j^{j+1} = -e^{-\lambda(j+1)} + e^{-\lambda j} \\ \text{b) } P(X > a) &= \int_a^{\infty} \lambda e^{-\lambda x} dx = \left[ -e^{-\lambda x} \right]_{x=a}^{x=\infty} = [0 + e^{-\lambda a}] = e^{-\lambda a} \\ \text{c) } P(X > a+b \mid X > a) &= \frac{P(X > a+b \text{ und } X > a)}{P(X > a)} = \frac{P(X > a+b)}{P(X > a)} = \frac{e^{-\lambda(a+b)}}{e^{-\lambda a}} = e^{-\lambda b} \end{aligned}$$

# Aufgabe 30

$$\begin{aligned} \text{a) } \int_0^2 \int_0^x Cxy \, dy \, dx &= C \cdot \int_0^2 \int_0^x xy \, dy \, dx = C \cdot \int_0^2 x \cdot \frac{1}{2} y^2 \Big|_0^x dx \\ &= \frac{1}{2} C \int_0^2 x^3 dx = \left( \frac{1}{8} x^4 \right) \Big|_0^2 = C \cdot 2 \Rightarrow C = \frac{1}{2} \\ \text{b) } f_X(x) &= \int_{-\infty}^{\infty} Cxy \cdot 1_{(0,x)}(y) \cdot 1_{(0,2)}(x) dy = Cx \int_0^x y \cdot 1_{(0,2)}(y) dy \\ &= Cx \cdot \int_0^x y dy = Cx \cdot \frac{y^2}{2} \Big|_0^x = \frac{C}{2} x^3 \cdot 1_{(0,2)}(x) \\ f_Y(y) &= \int_{-\infty}^{\infty} Cxy \cdot 1_{(0,x)}(y) \cdot 1_{(0,2)}(x) dx = Cy \int_y^{\infty} x \cdot 1_{(0,2)}(x) dx = Cy \cdot 1_{(0,2)}(y) \\ &\quad \cdot \int_y^2 x dx = Cy \cdot \left( \frac{x^2}{2} \right) \Big|_y^2 = Cy \cdot \left( 2 - \frac{y^2}{2} \right) = 2Cy - \frac{Cy^3}{2} \end{aligned}$$



$$\begin{aligned} \text{c) } P(X < 1, Y < 1) &= \int_0^1 \int_0^1 \frac{1}{4} xy \cdot 1_{(0,2)}(x) \cdot 1_{(0,2)}(y) dx dy = \int_0^1 \int_0^1 \frac{1}{4} xy \, dx dy = \frac{1}{16} \\ P(X=1, Y < 1) &= 0 = \int_{-\infty}^1 \int_1^1 f_{X,Y}(x,y) dy dx \\ P(X < 1, Y > \frac{1}{2}) &= \int_{-\infty}^1 \int_{\frac{1}{2}}^{\infty} f_{X,Y}(x,y) dy dx = \int_{\frac{1}{2}}^1 \int_{\frac{1}{2}}^x \frac{1}{4} xy \, dy dx = \dots = \frac{9}{256} \\ \hookrightarrow \frac{1}{2} < y < x < 1 \\ P(X < 1) &= \int_{-\infty}^1 f_X(x) dx = \int_{-\infty}^1 \int_{-\infty}^{\infty} f_{X,Y}(x,y) dy dx = \int_0^1 \frac{1}{4} x^3 dx = \dots = \frac{1}{16} \end{aligned}$$

# Aufgabe 31

$$\int_G f(x) dG = \int_H f(T(x)) \det D_T(x) dH$$

$$G = \{x \in \mathbb{R}^2 \mid \|x\|_2 \leq 2\} \quad T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$(r, \varphi) \mapsto \begin{pmatrix} r \cos \varphi \\ r \sin \varphi \end{pmatrix} \quad r \geq 0, \varphi \in [0, 2\pi)$$

$$H = \{(\varphi) \in \mathbb{R}^2 \mid 0 \leq r \leq 2, 0 \leq \varphi \leq 2\pi\}$$

$$\det D_T(r, \varphi) = \begin{vmatrix} \cos \varphi & -r \sin \varphi \\ \sin \varphi & r \cos \varphi \end{vmatrix} = r$$

$$f(x, y) = 1 + x^2 + y^2, \quad f(T(r, \varphi)) = 1 + r^2 \cos^2 \varphi + r^2 \sin^2 \varphi = 1 + r^2$$

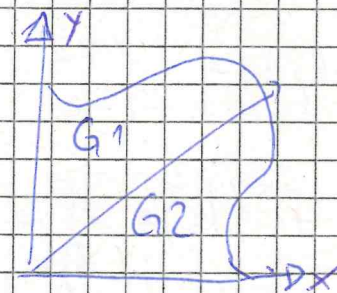
$$\int_G 1 + x^2 + y^2 dG = \int_H (1 + r^2) \cdot r \cdot dH = \int_0^{2\pi} \int_0^2 (1 + r^2) \cdot r \, dr d\varphi = \dots = 12\pi$$

# Aufgabe 32

$$G_1 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid 0 \leq x \leq 1, x \leq y \leq 1 - \frac{1}{10} \sin(2\pi x) \right\}$$

$$G_2 = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid 0 \leq y \leq 1, y \leq x \leq 1 - \frac{1}{10} \sin(2\pi y) \right\}$$

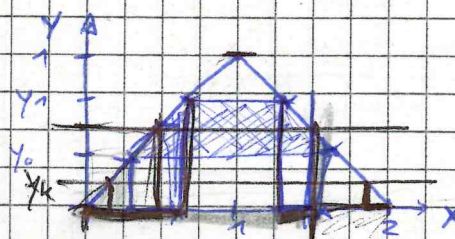
$$x = \underline{x}(y) \leq y \leq \bar{x}(y) = 1 - \frac{1}{10} \sin(2\pi y)$$



# Aufgabe 38

$$\text{a) } \text{Rechteckverteilung} \quad f(x) = \begin{cases} \frac{1}{1.5} & 0 \leq x \leq 1.5 \\ 0 & \text{sonst} \end{cases}$$

$$\text{b) } \Omega = [0, 1], \mathcal{A} \text{ die dazu passende Borel-}\sigma\text{-Algebra}$$



$$\text{c) } \{Y \in [y_0, y_1]\} = \{x \in \Omega \mid Y(x) \in [y_0, y_1]\} \mapsto P_Y^X(Y) := P(X^{-1}(Y)) = P(X \in Y)$$

$$y_0 := 1 - |x_0 - 1|, \quad y_1 := 1 - |x_1 - 1|$$

$$\{Y \in [y_0, y_1]\} = \{x \in \Omega \mid y_0 \leq x \leq y_1, y_2 - y_1 \leq x \leq 2 - y_0\}$$

$$\Rightarrow P_Y([y_0, y_1]) = \left( \int_{y_0}^{y_1} f(x) dx + \int_{2-y_1}^{2-y_0} f(x) dx \right) \cdot \frac{1}{1.5} = \frac{1}{1.5} (y_1 - y_0 + 2 - y_1 - (2 - y_0)) = \frac{1}{1.5} (y_1 - y_0)$$

$$P_Y([y_0, y_1]) = \begin{cases} 0 & \text{wenn } 2 - y_1 > 1.5 \\ \int_{y_0}^{1.5} f(x) dx & \text{wenn } 2 - y_0 \leq 1.5 \text{ und } 2 - y_1 > 1.5 \\ \int_{2-y_1}^{2-y_0} f(x) dx & \text{sonst} \end{cases}$$

$$\text{d) } F_Y(y) = \begin{cases} 0 & \text{falls } y < 0 \\ 2/3 y & 0 \leq y < 0.5 \\ 4/3 y - 1/3 & 0.5 \leq y < 1 \\ 1 & y \geq 1 \end{cases} \quad f_Y(y) = \frac{d}{dy} F(y) = \begin{cases} 2/3 & 0 \leq y < 0.5 \\ 4/3 & 0.5 \leq y < 1 \\ 0 & \text{sonst} \end{cases}$$



## Aufgabe 44

stetige Gleichverteilung

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{sonst} \end{cases}$$

$$f_x(x) = \begin{cases} 1 & -1 < x < 3 \\ 0 & \text{sonst} \end{cases}$$

$$f_y(y) = \begin{cases} 1/3 & 0 < y < 3 \\ 0 & \text{sonst} \end{cases}$$

$$f^{x+y}(z) = \int_{\mathbb{R}} f^x(x) f^y(z-x) dx \quad z \geq 0$$

$$= \int_{\mathbb{R}} \frac{1}{3} 1_{(-1,0)}(x) \cdot 1_{(0,3)}(z-x) dx$$

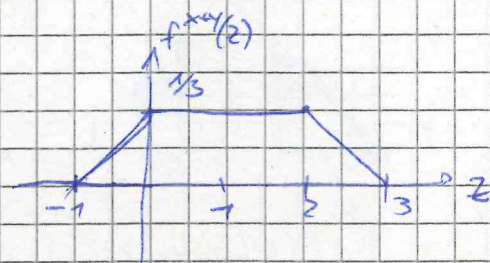
$$\left[ \text{NR: } 1_{(0,3)}(z-x) \neq 0 \text{ falls } 0 < z-x < 3 \Rightarrow -z < -x < 3-z \right.$$

$$\left. \begin{matrix} z > x > z-3 \\ \Rightarrow 1_{(0,3)}(z-x) = 1_{(z-3,z)}(x) \end{matrix} \right]$$

$$= \int_{\mathbb{R}} \frac{1}{3} 1_{(-1,0)} \cap (z-3,z)(x) dx \rightarrow \begin{matrix} -1 < x < 0, & z-3 < x < z \\ \Rightarrow -1 < z < 3 \end{matrix}$$

$$\left[ \begin{matrix} \rightarrow z \in (-1,0) & (-1,0) \cap (z-3,z) = (-1,z) \\ z \in [0,2] & (-1,0) \cap (z-3,z) = (-1,0) \\ z \in (2,3) & (-1,0) \cap (z-3,z) = (z-3,0) \end{matrix} \right]$$

$$= \begin{cases} \int_{-1}^z \frac{1}{3} dx = \frac{1}{3}(z+1) & z \in (-1,0) \\ \int_{-1}^0 \frac{1}{3} dx = \frac{1}{3} & z \in [0,2] \\ \int_{z-3}^0 \frac{1}{3} dx = \frac{1}{3}(3-z) & z \in (2,3) \\ 0 & \text{sonst} \end{cases}$$



## Aufgabe 46

$$X_i \sim \text{Exp}(\alpha) \quad f_{X_i}^{X_i}(x) = \alpha \cdot e^{-\alpha x} \cdot 1_{(0,\infty)}(x) \quad \forall i=1, \dots, n$$

$$\text{Beh: } \bar{Y}_n = \sum_{i=1}^n X_i \sim \Gamma(\alpha, n)$$

Bew  $n=1$  nichts zu zeigen

$$n=2 \quad \bar{Y}_2 = \bar{X}_1 + \bar{X}_2$$

$$\int \bar{Y}_2(x) = f^{X_1+X_2}(x) = \int_0^x f^{X_1}(z) f^{X_2}(x-z) dz$$

$$= \int_0^x \alpha e^{-\alpha z} \cdot \alpha e^{-\alpha(x-z)} dz = \int_0^x \alpha^2 e^{-\alpha x} dz = \alpha^2 x \cdot e^{-\alpha x}$$

$$= \frac{\alpha^2}{\Gamma(2)} x^{2-1} e^{-\alpha x}$$

$$n \rightarrow n+1 \quad \bar{Y}_{n+1} = \sum_{i=1}^n X_i + X_{n+1} = \bar{Y}_n + X_{n+1}$$

$$f^{\bar{Y}_{n+1}}(x) = f^{\bar{Y}_n + X_{n+1}}(x) = \int_0^x f^{\bar{Y}_n}(z) f^{X_{n+1}}(x-z) dz$$

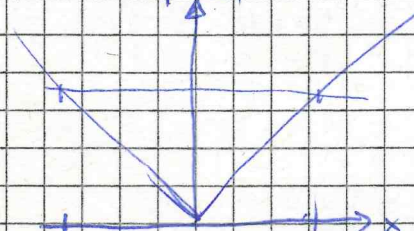
$$= \int_0^x \frac{\alpha^n}{\Gamma(n)} z^{n-1} \cdot e^{-\alpha z} \cdot \alpha e^{-\alpha(x-z)} dz = \frac{\alpha^{n+1}}{\Gamma(n)} e^{-\alpha x} \int_0^x z^{n-1} dz$$

$$= \frac{\alpha^{n+1}}{\Gamma(n+1)} x^n e^{-\alpha x}$$

## Aufgabe 52

$$a) \quad X \rightarrow |X| \quad Y$$

$$Y = |X|$$



$$Y = |X| = \begin{cases} X & \text{falls } X \geq 0 \\ -X & \text{falls } X < 0 \end{cases}$$

$$F(y) = F_x(-X \leq Y \leq X)$$

$$P(Y \leq x) = P(|X| \leq x) = P(-x \leq X \leq x)$$

$$\frac{d}{dx} \int_{-x}^x f_x(x) dx = \frac{d}{dx} \int_{-x}^x \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx = \frac{d}{dx} \left[ \frac{1}{\sqrt{2\pi}} \int_{-x}^x \exp\left(-\frac{u^2}{2}\right) \frac{1}{\sqrt{2\pi}} du \right]$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$b) \quad P(-1 \leq X \leq 1 \cap |X| > 1) \neq P(-1 \leq X \leq 1) \cdot P(|X| > 1) > 0$$

$$\Rightarrow X \text{ und } |X| \text{ stoch. abh.}$$

$$c) \quad E|X| = \int_{-\infty}^{\infty} x f^x(x) dx = \int_0^{\infty} x \cdot \sqrt{\frac{2}{\pi}} e^{-\frac{x^2}{2}} dx = \int_0^{\infty} \sqrt{\frac{2}{\pi}} e^{-u} du = \sqrt{\frac{2}{\pi}}$$

$$\text{Var}(|X|) = E(|X|^2) - (E(|X|))^2 = E(X^2) - (E(|X|))^2 = \text{Var}(X) + (EX)^2 - \frac{2}{\pi} = 1 - \frac{2}{\pi} = \frac{\pi-2}{\pi}$$

## Aufgabe 45

$$a) \quad p=0,1, \text{ Taktlänge} = 1 \text{ Min}$$

$$Z_i = \text{Ankunft eines Kunden im } i\text{-ten Takt} \quad Z_i \sim B(p), i=1, \dots, 60$$

 $\bar{Z}$  Anzahl der ankommenden Kunden in einer Stunde

$$\bar{Z} = \sum_{i=1}^{60} Z_i, \quad \bar{Z} \sim B(n, p) = B(60, 0,1)$$

$$\text{Dichte } f^{60}(h) = \binom{60}{h} (0,1)^h (0,9)^{60-h}$$

$$b) \text{ Normalverteilung } N(\mu, \sigma^2) = N(np, np(1-p)) = N(6, 5,4)$$

$$\text{1. Faustregel } n \cdot p(1-p) \geq 9$$

$$\text{2. Faustregel } n \cdot p \geq 5, \quad n(1-p) \geq 5$$

$$c) \quad P(X \leq 3) = \sum_{k=0}^3 f^{60}(k) = 0,137 \quad \left[ \Phi\left(\frac{3-6}{\sqrt{5,4}}\right) = 0,141 \right]$$

$$\approx \Phi\left(\frac{3-6}{\sqrt{5,4}}\right) = \Phi\left(\frac{-3}{2,324}\right) = \Phi(-1,29) = 1 - \Phi(1,29)$$

$$= 1 - 0,9015 = 0,0985$$



# Aufgabe 53

a)	$f_{(X,Y)}(j,k)$	$\bar{X} = j$		$P(Y=k)$
		-1	1	
	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
	1	$\frac{1}{8}-c$	$\frac{1}{8}-c$	$\frac{1}{4}$
	2	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
	$P(X=j)$	$\frac{1}{2}+c$	$\frac{1}{2}-c$	1

$$b) P(Y=k | X=1) = \frac{P(Y=k, X=1)}{P(X=1)} = \begin{cases} \frac{1/8}{1/2-c} & k=0 \\ \frac{1/8-c}{1/2-c} & k=1 \\ \frac{1/4}{1/2-c} & k=2 \end{cases}$$

$$c) EX = \sum_{j=-1}^1 j \cdot P(X=j) = (-1) \cdot \left(\frac{1}{2}+c\right) + 1 \cdot \left(\frac{1}{2}-c\right) = -2c$$

$$EY = \sum_{k=0}^2 k \cdot P(Y=k) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{2} = 5/4$$

$$E(XY) = \sum_{j,k} j \cdot k \cdot P(X=j, Y=k) = (-1) \cdot 0 \cdot \left(\frac{1}{8}+c\right) + (-1) \cdot 1 \cdot \left(\frac{1}{8}-c\right) + (-1) \cdot 2 \cdot \frac{1}{4} + 1 \cdot 0 \cdot \left(\frac{1}{8}-c\right) + 1 \cdot 1 \cdot \left(\frac{1}{8}-c\right) + 1 \cdot 2 \cdot \frac{1}{4} = -2c$$

$$d) \text{Kov}(X,Y) = E(XY) - EX EY = -2c - (-2c) \cdot 5/4 = (1 - 5/4)(-2c) = \frac{c}{2}$$

Für  $c=0$  sind  $X$  und  $Y$  unkorreliert

# Aufgabe 54

$$X \sim \text{Exp}(\lambda), Y \sim \text{Exp}(\lambda)$$

$$EX = EY = 1/\lambda, \text{Var}(X) = \text{Var}(Y) = 1/\lambda^2$$

$$E(S) = E(2Y+X) = 3/\lambda, \text{Var}(S) = \text{Var}(2Y+X) = 5/\lambda^2$$

$$\text{Kov}(S,D) = \frac{1}{2} \text{Var}(S+D) - \frac{1}{2} \text{Var}(S) - \frac{1}{2} \text{Var}(D) = \frac{1}{2} (16 \text{Var}(Y) - 2 \frac{5}{\lambda^2}) = \frac{3}{\lambda^2}$$

# Aufgabe 61

$$T \sim \mathcal{L}(10), ET = \frac{11}{2}, \text{Var}[T] = \frac{99}{12}$$

$$Y_i = \mathcal{B}(5; 0,2) \quad i=1, \dots, 10 \quad EY_i = 1, \text{Var}[Y_i] = \frac{4}{5}$$

$$Z = \sum_{i=1}^T Y_i \quad "Z \sim \mathcal{B}(T \cdot 5; 0,2)"$$

$$P(Z=k | T=t) = P(Z_k=k | T=t) = b(t \cdot 5; 0,2; k)$$

$$= P(Z=k, T=t) \cdot \frac{1}{P(T=t)}$$

$$\Rightarrow P(Z=k, T=t) = P(Z=k | T=t) \cdot P(T=t) = b(t \cdot 5; 0,2; k) \cdot \frac{1}{10}$$

$$b) E(Z) = E[T] \cdot E[Y_1] = \frac{11}{2} \cdot 1 = \frac{11}{2}$$

$$\text{Var}(Z) = \sum_{t=1}^{10} \frac{1}{10} E[Z | T=t]^2 = \sum_{t=1}^{10} \frac{t^2}{10} = \frac{110}{2} \cdot \frac{1}{10} = \frac{11}{2}$$

# diskret: $P(Z=k) = f(k)$

$$E[Z | T=t] = t \cdot 5 \cdot 0,2 = t$$

$$\sim \mathcal{B}(t \cdot 5; 0,2)$$

$$\text{Var}[Z] = E[T] \cdot \text{Var}[Y_1] + \text{Var}[T] \cdot E[Y_1]^2$$

$$= \frac{11}{2} \cdot \frac{4}{5} + \frac{99}{12} \cdot 1^2 = \dots = 126,5 \cdot \frac{1}{10} = 12,65$$

$$\text{Var}[Z | T=t] = t \cdot 5 \cdot \frac{4}{5} = t$$

# Aufgabe 60

$$X \sim \mathcal{N}(0, I_n), X_i \sim \mathcal{N}(0,1) \quad i=1, \dots, n$$

$$Y = a + AX, Y \sim \mathcal{N}(a, K) \quad a \in \mathbb{R}^n, A \in \mathbb{R}^{n \times n}$$

$$K = A \cdot A^T \quad k_{ij} = \text{Kov}(Y_i, Y_j) = \text{Kov}(Y_j, Y_i) = k_{ji}$$

$$X = \begin{pmatrix} 3 & -a & b \\ b & a & 1 \\ a & b & -4 \end{pmatrix} \begin{pmatrix} G_1 \\ G_2 \\ G_3 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = Y$$

$$\begin{pmatrix} 3 & -a & b \\ b & a & 1 \\ a & b & -4 \end{pmatrix} \begin{pmatrix} 3 & b & a \\ -a & a & b \\ b & 1 & -4 \end{pmatrix} = \begin{pmatrix} 10+a^2 & 2b-a \\ 4b-a^2 & b^2+a^2+1 \\ 3a-ab-4b & 2ab-4 \end{pmatrix} X_1$$

$$\begin{pmatrix} 3 & -a & b \\ b & a & 1 \\ a & b & -4 \end{pmatrix} \begin{pmatrix} 3 & b & a \\ -a & a & b \\ b & 1 & -4 \end{pmatrix} = \begin{pmatrix} 9+a^2+b^2 & 4b-a^2 & 3a-ab-4b \\ 4b-a^2 & b^2+a^2+1 & 2ab-4 \\ 3a-ab-4b & 2ab-4 & a^2+b^2+16 \end{pmatrix} X_2, X_3$$

$$f^X(x) = \frac{1}{\sqrt{\det(K)}} \left( \frac{1}{\sqrt{2\pi}} \right)^n e^{-\frac{1}{2}(x-a)^T (K^{-1})(x-a)} \quad x \in \mathbb{R}^n$$

$$\det K = (9+a^2+b^2) \dots$$

$$2ab-4 = 4b-a^2 = 3a-ab-4b = 0$$

$$ab=2, b = a^2/4$$

$$a: a^2/4 = 2$$

$$a^2 = 8 \Rightarrow a = \pm 2 \Rightarrow b = 1$$

$$X_1 = 3G_1 + 2G_2 + G_3 + 2 = \mathcal{N}(2, 14)$$

$$X_2 = G_1 + 2G_2 + G_3 + 1 = \mathcal{N}(1, 5)$$

$$X_3 = 2G_1 + G_2 - 4G_3 = \mathcal{N}(0, 21)$$



Bsp. am Ende von 7.6

$X$  ZVektor  $X=(X_1, X_2)$ , dichte ist konstant auf  $M=\{x \in \mathbb{R}^2 \mid \|x\|_2^2 \leq R^2\}$

$f^*(x) = \frac{1}{\pi R^2} \cdot 1_{(M)}(x) = \frac{1}{\pi R^2} \cdot 1_{(M)}(x)$

Randdichte

$f^{X_1}(x_1) = \int_{\mathbb{R}} f^*(x_1, x_2) dx_2 = \int_{-\sqrt{R^2-x_1^2}}^{\sqrt{R^2-x_1^2}} \frac{1}{\pi R^2} dx_2 = \frac{2\sqrt{R^2-x_1^2}}{\pi R^2}$

$f^{X_1}(x_1) = 0$  für  $x_1 \in \mathbb{R} \setminus (-R, R)$

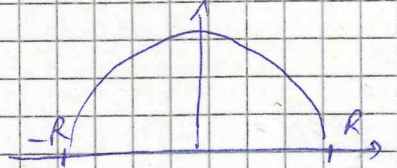
$f^{X_2}(x_2) = \frac{2\sqrt{R^2-x_2^2}}{\pi R^2} \cdot 1_{(-R, R)}(x_2)$

$EX_1 = \int_{\mathbb{R}} f^{X_1}(x_1) \cdot x_1 \cdot dx_1 = \int_{-R}^R \frac{2\sqrt{R^2-x_1^2}}{\pi R^2} x_1 dx_1 = 0 = EX_2$  (analog zu  $X_1$ )

$Kor(X_1, X_2) = E(X_1 \cdot X_2) - EX_1 \cdot EX_2$

$E[X_1 \cdot X_2] = \int_{\mathbb{R}^2} x_1 \cdot x_2 \cdot f^*(x_1, x_2) d(x_1, x_2) = \frac{1}{\pi R^2} \int_{-R}^R x_1 \left[ \int_{-\infty}^{\infty} x_2 \cdot 1_{(M)}(x) dx_2 \right] dx_1 = 0$

$\Rightarrow Kor(X_1, X_2) = 0 \Rightarrow X_1, X_2$  sind unkorreliert



Bsp

Wartezeiten: Ankomst von Kunden  $Y$

$Y \sim \text{geo}^+(p)$ ,  $EY = \frac{1}{p}$ ,  $\text{Var}[Y] = \frac{1-p}{p^2}$

$T_i \sim \text{geo}^+(\bar{p})$ ,  $ET_i = \frac{1}{\bar{p}}$ ,  $\text{Var}[T_i] = \frac{1-\bar{p}}{\bar{p}^2}$

$ES = EY \cdot ET_1 = \frac{1}{p} \cdot \frac{1}{\bar{p}}$

$\text{Var } S = EY \cdot \text{Var } T_1 + \text{Var } Y \cdot (E[T_1])^2$

$= \frac{1}{p} \cdot \frac{1-\bar{p}}{\bar{p}^2} + \frac{1-p}{p^2} \cdot (1-\bar{p}) \cdot \frac{1}{\bar{p}^2} = \frac{1-p}{p^2 \bar{p}^2} + \frac{1-p}{p^2 \bar{p}^2} = \frac{1-p}{p^2 \bar{p}^2}$

$S_n = \sum_{i=1}^n T_i \sim \text{Nb}(n, p\bar{p})$

$P(S=k) = \sum_n P(Y=n) \cdot P(S_n=k) = \sum_n p(1-p)^{n-1} \binom{k-1}{n-1} \bar{p}^n (1-\bar{p})^{k-n}$

$= p\bar{p} \sum_{n=1}^k \binom{k-1}{n-1} (\bar{p}-p\bar{p})^{n-1} (1-\bar{p})^{k-n} = p\bar{p}(1-p\bar{p})$

P 68

$X \sim R[105, 135]$   $X_i$  st. unabh. identisch verteilt

max. Ladung 7800 kg,  $S_{64} = \sum_{i=1}^{64} X_i$

$\frac{S_n - ES_n}{\text{Std } S_n} \sim N(0, 1)$

$EX_i = \frac{105+135}{2}$

$\text{Var } X_i = \frac{(135-105)^2}{12}$

a)  $ES = EY \cdot EX_1 = EY \cdot \frac{105+135}{2} = 1200 \cdot 120$

$\text{Var } S = EY \cdot \text{Var } X_1 + \text{Var } Y \cdot (EX_1)^2 = \text{Var } X_1 = \frac{30^2}{12}$

b)  $7560 \leq S_{64} \leq 7800$

$\frac{S_{64} - ES_{64}}{\sqrt{\text{Var } S_{64}}} \sim N(0, 1) \rightarrow \frac{S_{64} - ES_{64}}{\sqrt{\text{Var } S_{64}}} \sim \sqrt{\text{Var } S_{64}} \cdot N(0, 1)$

$ES_{64} = 64 \cdot 120 = 7680$

$\text{Var } S_{64} = 64 \cdot \frac{30^2}{12} = 64 \cdot 75$

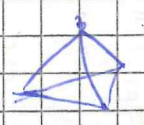
$P(7560 \leq S_{64} \leq 7800) = P(S_{64} \leq 7800) - P(S_{64} \leq 7560)$

$= P(-120 \leq S_{64} - ES_{64} \leq 120) = P(|S_{64} - ES_{64}| \leq 120)$

$\left[ \begin{aligned} Y = S - ES, \quad P(|S - ES| \geq \epsilon) &\leq \frac{1}{\epsilon^2} E[(S - ES)^2] \\ \Rightarrow 1 - P(|S - ES| \geq \epsilon) &\geq 1 - \frac{1}{\epsilon^2} \text{Var } S \end{aligned} \right]$

$= 1 - P(|S_{64} - ES_{64}| \geq 120) \geq 1 - \frac{64 \cdot 75}{120^2} = 1 - \frac{1}{3} = \frac{2}{3}$

c)  $P(S_{64} \leq 7800) = P\left(\frac{S_{64} - ES_{64}}{\sqrt{\text{Var } S_{64}}} \leq \frac{120}{\sqrt{64 \cdot 75}}\right) = \Phi(1,73) = 0,9573$



P 69

Nachfrage:  $X \sim \text{Exp } \lambda$ ,  $EX = \frac{1}{\lambda}$ ,  $\text{Var } X = \frac{1}{\lambda^2}$

$q$  = Tagesverbrauch,  $p_1$  = Einkaufspreis,  $p_2$  = Verkaufspreis / Einheit

a)  $G(x) = p_2 \min(x, q) - p_1 \cdot q$  = erwarteter Gewinn - Y

$g(q) = EY = EG(x) = \int_{\mathbb{R}} G(x) \cdot f^*(x) dx$

$= \int p_2 \min(x, q) \cdot f(x) dx - p_1 q \underbrace{\int f^*(x) dx}_{=1}$



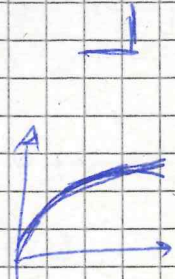
$$= p_2 \int_0^{\infty} \min(x, q) \cdot \lambda e^{-\lambda x} dx - p_1 q$$

$$\text{NR} \int_0^{\infty} \min(x, q) e^{-\lambda x} \cdot \lambda dx = \int_0^q x \lambda e^{-\lambda x} dx + \int_q^{\infty} q \cdot e^{-\lambda x} \lambda dx$$

$$= \frac{1}{\lambda} (1 - e^{-\lambda q})$$

$$= \frac{p_2}{\lambda} (1 - e^{-\lambda q}) - p_1 q$$

$$\text{und } g'(q) = p_2 \cdot e^{-\lambda q} - p_1 \stackrel{!}{=} 0 \rightarrow q = \frac{1}{\lambda} \ln\left(\frac{p_2}{p_1}\right)$$



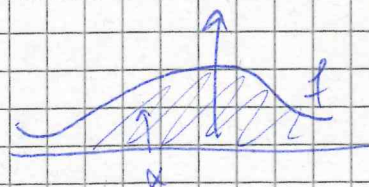
$$F(x) = P(X \leq x) = 1 - e^{-\lambda x} = p$$

$$\Rightarrow x = F^{-1}(p) = -\frac{1}{\lambda} \ln(1-p)$$

$$\lambda = \frac{1}{1000} \quad p = 0,025$$

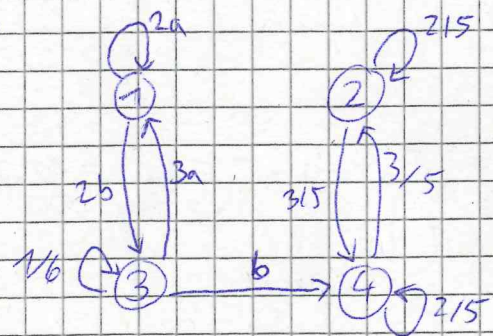
$$U_{2,5\%} = F^{-1}(0,025) = -\frac{1}{1000} \ln(1-0,025) \approx 25,32$$

$$U_{95\%} = F^{-1}(0,95) = -\frac{1}{1000} \ln(1-0,95) \approx 295,57$$



P74

a)



$$\begin{aligned} 2a + 2b &= 1 \\ 5a &= 1 \end{aligned} \Rightarrow \begin{aligned} a + b &= 1/2 \\ a &= 1/5 \end{aligned}$$

$$\begin{aligned} a + b &= 1/2 = 6/12 \\ 3a + b &= 5/6 = 10/12 \\ 4a + 2b &= 16/12 \\ 2a + b &= 8/12 = 2/3 = 4/6 \\ a + b &= 1/2 = 3/6 \end{aligned}$$

$$\boxed{a = 1/6, b = 1/3}$$

$$b) \pi = \pi P \Rightarrow \pi(P-I) = 0$$

$$P-I = \begin{pmatrix} 1/3-1 & 0 & 2/5 & 0 \\ 0 & 2/5-1 & 0 & 3/5 \\ 1/2 & 0 & 1/6-1 & 1/3 \\ 0 & 3/5 & 0 & 2/5-1 \end{pmatrix}$$

$$P^T - I = \begin{pmatrix} 2/3 & 0 & 0 & 0 \\ 0 & 3/5 & 0 & 0 \\ 2/3 & 0 & -5/6 & 0 \\ 0 & 3/5 & 1/3 & -2/5 \end{pmatrix}$$

$$\frac{2}{3}x_1 - \frac{5}{6}x_3 = -\frac{2}{3}x_1 + \frac{1}{2}x_3$$

$$P^T - I) \pi = 0$$

$$\begin{pmatrix} -2/3 & 0 & 1/2 & 0 & 0 \\ 0 & -3/5 & 0 & 3/5 & 0 \\ 2/3 & 0 & -5/6 & 0 & 0 \\ 0 & 3/5 & 1/3 & -3/5 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1/2-5/6 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 \\ 2/3 & 0 & -5/6 & 0 & 0 \\ 0 & 3/5 & 1/3 & -3/5 & 0 \end{pmatrix}$$

$$x_1 = x_3 = 0$$

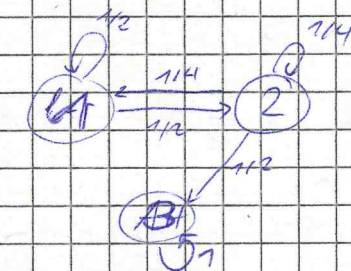
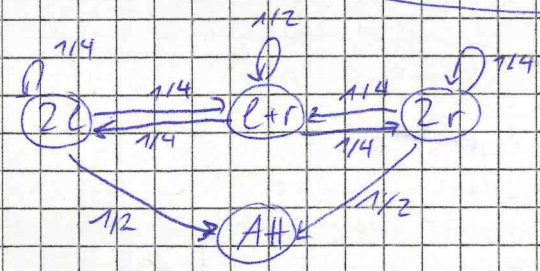
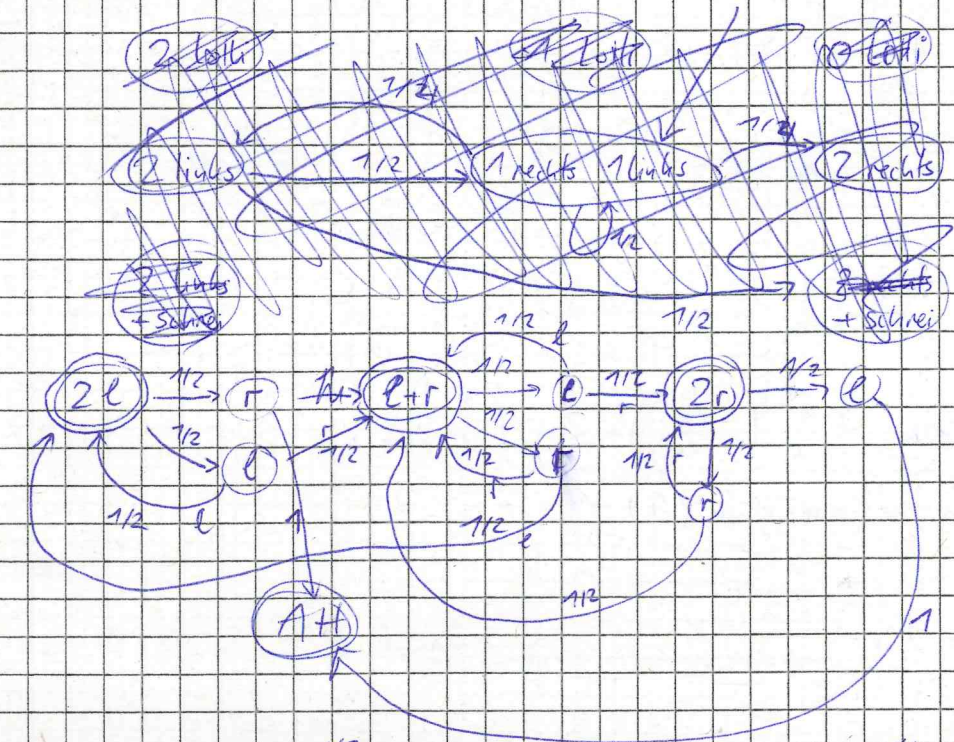
$$x_2 = x_4$$

$$\pi = \begin{pmatrix} 0 \\ 1/2 \\ 0 \\ 1/2 \end{pmatrix}$$

$$\pi = v^T \text{ mit } \pi_i \geq 0 \text{ und } \sum_{i=1}^k \pi_i = 1 \Rightarrow \mu = 1/2$$

$$\text{die GG V lautet } \pi = (0, \frac{1}{2}, 0, \frac{1}{2})$$

(15)



$$P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 1/4+1/8 & 1/4+1/8 & 1/4 \\ 1/8+1/16 & 1/8+1/16 & 1/2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 1/3 & 0 & 1/3 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 1/2 & 0 & 1/4 & 0 & 1/4 & 0 \\ 0 & 1/2 & 0 & 1/4 & 0 & 1/4 \\ 1/4 & 0 & 1/4 & 0 & 0 & 1/2 \\ 0 & 1/4 & 0 & 1/2 & 0 & 1/4 \\ 1/4 & 0 & 0 & 1/2 & 1/4 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



$$\sum^{(u)} = (1, 0, 0, 0, 0, 0)$$

$$\sum^{(2u)} = \sum^{(u)} \otimes \sum^{(u)} = \sum^{(u)} (P^*)$$

$$\sum^{(4)} = (1/2, 0, 1/4, 0, 1/4, 0)$$

$$\sum^{(4)} = (3/8, 0, 3/16, 0, 3/16, 1/4)$$

$$\sum^{(6)} = (9/32, 0, 9/64, 0, 9/64, 3/16)$$

$$\sum^{(2^{k-1}), (2^k)} = \left( \left(\frac{1}{2}\right)^{k-1}, 0, \frac{1}{4} \left(\frac{1}{2}\right)^{k-1}, 0, \frac{1}{4} \left(\frac{1}{2}\right)^{k-1}, 1 - \left(\frac{1}{2}\right)^{k-1} \right)$$

$$\lim_{k \rightarrow \infty} \sum^{(2^k)} = (0, 0, 0, 0, 0, 1)$$

P79

$X_1, X_n$  unabhängig identisch Poisson verteilt mit  $\lambda > 0$

$$L(x_1, \dots, x_n; \lambda) = ? \quad e^{-\lambda} \frac{\lambda^k}{k!} = f(k) \quad \text{Poisson}$$

$$L(x_1, \dots, x_n; \lambda) = \prod_{i=1}^n f(x_i) = e^{-\lambda n} \cdot \frac{\lambda^{\sum_{i=1}^n x_i}}{\prod_{i=1}^n x_i!}$$

$\bar{x}$  = arithmetisches Mittel

$$= e^{-\lambda n} \cdot \lambda^{n \cdot \bar{x}} \cdot \frac{1}{\prod_{i=1}^n x_i!}$$

$$\ln(L(x_1, \dots, x_n; \lambda)) = -\lambda n + \ln(\lambda^{n \cdot \bar{x}}) + \ln\left(\frac{1}{\prod_{i=1}^n x_i!}\right)$$

$$= -\lambda n + (n \cdot \bar{x}) \ln(\lambda) +$$

$$\ln(L(x_1, \dots, x_n; \lambda))' = -n + \frac{n \cdot \bar{x}}{\lambda} \stackrel{!}{=} 0$$

$$n \cdot \bar{x} \stackrel{!}{=} n \cdot \lambda \Rightarrow \lambda = \bar{x}$$

Erwartungstreue?

$$E(\hat{\theta}) = \theta$$

$$E\left(e^{-\bar{x}} \frac{\bar{x}^k}{k!}\right) = \bar{x}$$

$$E(\hat{\lambda}) = E\left(\frac{1}{n} \sum_{i=1}^n X_i\right) = \frac{1}{n} E\left(\sum_{i=1}^n X_i\right) = \frac{1}{n} E(n \cdot \lambda) = \lambda$$

P80

0,329

0,324

0,334

normalverteilt, Konfidenzintervalle gesucht

0,339

0,328

0,338

EW =  $\mu$ , Var =  $\sigma^2$

0,331

0,327

0,326

Niveau  $1 - \alpha = 0,95$

$$P(G_1 < \theta < G_2) = 0,95$$

$$X \sim N(\mu, \sigma^2) \quad \frac{\sqrt{n}(\bar{X} - \mu)}{s} \sim t_{n-1}$$

$$\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 \sim \chi_{n-1}^2$$

$$\bar{x} = 0,3304, s^2 = 2,5 \cdot 10^{-5}, s = 5 \cdot 10^{-3}$$

$$P81) X = 215, n = 17 \quad X \sim N(\mu, \sigma^2) \quad \sigma^2 = 2$$

$$\text{Konfidenzschätzung von } \mu \text{ bei bekannter Varianz: } \frac{\sqrt{n}(\bar{X} - \mu)}{\sigma} \sim N(0,1)$$

$$\text{Ziel: } P(\mu_n < \mu < \mu_o) = 1 - \alpha$$

$$P\left(-z_{1-\frac{\alpha}{2}} < \frac{\sqrt{n}(\bar{X} - \mu)}{\sigma} < z_{1-\frac{\alpha}{2}}\right) = 1 - \alpha$$

$$\Rightarrow P\left(\bar{X} - z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha$$

$$\alpha = 0,1 \Rightarrow z_{1-\frac{\alpha}{2}} = z_{0,95} = 1,64 \quad (\text{Quantile } N(0,1))$$

$$\alpha = 0,02 \Rightarrow z_{1-\frac{\alpha}{2}} = z_{0,99} = 2,33$$

$$\text{KI: } \left[215 - z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{2}{17}}, 215 + z_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{2}{17}}\right]$$

P80

KI für  $\mu$  bei unbekannter Varianz

$$\frac{\sqrt{n}(\bar{X} - \mu)}{s} \sim t_{n-1} \quad n=9, \alpha=0,05$$

Student-t-Vert.

$$\text{KI: } \left[\bar{x} - t_{1-\frac{\alpha}{2}; n-1} \cdot \frac{s}{\sqrt{n}}, \bar{x} + t_{1-\frac{\alpha}{2}; n-1} \cdot \frac{s}{\sqrt{n}}\right] = \left[0,3304 - 2,31 \cdot \frac{5 \cdot 10^{-3}}{3}; 0,3304 + 2,31 \cdot \frac{5 \cdot 10^{-3}}{3}\right]$$

$$t_{1-\frac{\alpha}{2}; n-1} = t_{0,975; 8} = 2,31$$

Varianz einer nv-ZV:

$$\chi_{\frac{\alpha}{2}; n-1}^2 \leq \frac{(n-1) \cdot s^2}{\sigma^2} \leq \chi_{1-\frac{\alpha}{2}; n-1}^2$$

$$\text{KI: } \sigma^2 \in \left[\frac{(n-1) s^2}{\chi_{1-\frac{\alpha}{2}; n-1}^2}, \frac{(n-1) s^2}{\chi_{\frac{\alpha}{2}; n-1}^2}\right] = \left[\frac{8 \cdot 2,5 \cdot 10^{-5}}{17,5}, \frac{8 \cdot 2,5 \cdot 10^{-5}}{2,48}\right]$$