$\mathcal{ALC}$

https://wwwcip.cs.fau.de/~oc45ujef/ai/alc.tex

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Syntax

$$F_{\mathcal{ALC}} ::= C \mid \top \mid \bot \mid \overline{F_{\mathcal{ALC}}} \mid F_{\mathcal{ALC}} n \sqcup F_{\mathcal{ALC}} \mid F_{\mathcal{ALC}} \sqcap F_{\mathcal{ALC}} \mid \exists \mathrm{R}.\ F_{\mathcal{ALC}} \mid \forall \mathrm{R}.\ F_{\mathcal{ALC}}$$

Semantics

The semantics of $\mathcal{ALC}$ is a tuple $\langle \mathcal{D}, [\![-]\!] \rangle$ where $\mathcal{D}$ is a non-empty Set and $[\![-]\!]$ a mapping into $\mathcal{P}(\mathcal{D})$

$$\begin{aligned}
[\![\bot]\!] &= \emptyset \\
[\![\top]\!] &= \mathcal{D} \\
[\![c]\!] &= D \subseteq \mathcal{D} \\
[\![\overline{\varphi}]\!] &= \mathcal{D} \setminus [\![\varphi]\!] \\
[\![\varphi \sqcap \psi]\!] &= [\![\varphi]\!] \cap [\![\psi]\!] \\
[\![\varphi \sqcup \psi]\!] &= [\![\varphi]\!] \cup [\![\psi]\!] \\
[\![\exists \mathrm{R}.\varphi]\!] &= \{ d \in \mathcal{D} \mid \exists y.\ xRy \wedge y \in [\![\varphi]\!] \} \\
[\![\forall \mathrm{R}.\varphi]\!] &= \{ d \in \mathcal{D} \mid \forall y.\ xRy \implies y \in [\![\varphi]\!] \}
\end{aligned}$$

Tableau calculus

$$\frac{\begin{array}{c} x : c \\ x : \overline{c} \end{array}}{\bot} \qquad \frac{x : \exists \mathrm{R}.\ \varphi}{\begin{array}{c} x \mathrm{R} y \\ y : \varphi \end{array}}$$

$$\frac{x : \varphi \sqcap \psi}{\begin{array}{c} x : \varphi \\ x : \psi \end{array}}$$

$$\frac{x : \varphi \sqcup \psi}{x : \varphi \quad \mid \quad x : \psi}$$

$$\frac{\begin{array}{c} x : \forall \mathrm{R}.\ \varphi \\ x \mathrm{R} y \end{array}}{y : \varphi}$$