AI I Calculi Cheatsheet

1 Natural Deduction

$$\frac{A \quad B}{A \wedge B} \wedge \mathrm{I}$$

$$\frac{A \wedge B}{A} \wedge \mathrm{E_l}$$

$$\frac{A \wedge B}{B} \wedge \mathrm{E_r}$$

$$\frac{A}{A \vee B} \vee \mathrm{I_l}$$

$$\frac{B}{A \vee B} \vee \mathrm{I_r}$$

$$\frac{A \vee B \quad \overset{[A]}{C} \quad \overset{[B]}{C}}{C} \vee \mathrm{E}$$

$$\frac{A \Longrightarrow B \quad A}{B} \Longrightarrow \mathrm{E}$$

$$\frac{\overset{[A]}{B}}{A \Longrightarrow B} \Longrightarrow \mathrm{I}$$

$$\frac{}{A \vee \neg A} \mathrm{TND}$$

$$\frac{\overset{[A]}{C} \quad \overset{[A]}{\neg C}}{\neg A} \neg \mathrm{I}$$

$$\frac{\neg A \quad A}{\bot} \bot \mathrm{I}$$

$$\frac{\bot}{A} \bot \mathrm{E}$$

First-Order

$$\frac{A}{\forall X.\ A} \forall \mathrm{I}\ ^0$$

$$\frac{\forall X.\ A}{[B/X]A} \forall \mathrm{E}$$

$$\frac{[B/X]A}{\exists X.\ A} \exists \mathrm{I}$$

$$\frac{\exists X.\ A \quad \overset{[[c/X]A]}{C} \quad c\ \text{fresh}}{C} \exists \mathrm{E}$$

^{0}where A does not depend on a context with X as a free variable.

2 Tableau

$$\frac{A \wedge B^{\mathrm{T}}}{\begin{array}{c} A^{\mathrm{T}} \\ B^{\mathrm{T}} \end{array}}$$

$$\frac{A \wedge B^{\mathrm{F}}}{A^{\mathrm{F}} \quad | \quad B^{\mathrm{F}}}$$

$$\frac{\neg A^{\mathrm{T}}}{A^{\mathrm{F}}}$$

$$\frac{\neg A^{\mathrm{F}}}{A^{\mathrm{T}}}$$

$$\frac{\begin{array}{c} A^{\alpha} \\ A^{\beta} \end{array}}{\bot}\ \alpha \neq \beta$$

$$\frac{A \Longrightarrow B^{\mathrm{T}}}{A^{\mathrm{F}} \quad | \quad B^{\mathrm{T}}}$$

$$\frac{A \Longrightarrow B^{\mathrm{F}}}{\begin{array}{c} A^{\mathrm{T}} \\ B^{\mathrm{F}} \end{array}}$$

$$\frac{A \vee B^{\mathrm{T}}}{A^{\mathrm{T}} \quad | \quad B^{\mathrm{T}}}$$

$$\frac{A \vee B^{\mathrm{F}}}{\begin{array}{c} A^{\mathrm{F}} \\ B^{\mathrm{F}} \end{array}}$$

Free variable Tableau

$$\frac{\forall X.\ A^{\mathrm{T}} \quad Y\ \text{new}}{[Y/X]A^{\mathrm{T}}}$$

$$\frac{\forall X.\ A^{\mathrm{F}} \quad \{X_1,...,X_k\} = \mathrm{free}(\forall X.\ A) \quad f \in \Sigma_k^{\mathrm{sk}}\ \text{new}}{([f(X_1,...,X_k)/X]A)^{\mathrm{F}}}$$

$$\frac{\exists X.\ A^{\mathrm{F}} \quad Y\ \text{new}}{[Y/X]A^{\mathrm{F}}}$$

$$\frac{\exists X.\ A^{\mathrm{T}} \quad \{X_1,...,X_k\} = \mathrm{free}(\exists X.\ A) \quad f \in \Sigma_k^{\mathrm{sk}}\ \text{new}}{([f(X_1,...,X_k)/X]A)^{\mathrm{T}}}$$

$$\frac{\begin{array}{c} A^{\alpha} \\ B^{\beta} \end{array} \quad \alpha \neq \beta \quad \sigma(A) = \sigma(B)}{\bot : \sigma}$$

3   Resolution

$$\frac{A^{\mathrm{T}}\vee C \qquad B^{\mathrm{F}}\vee D \qquad \sigma=\mathrm{mgu}(A,B)}{\sigma(C)\vee\sigma(D)}$$

$$\frac{A^{\alpha}\vee B^{\alpha}\vee C \qquad \sigma=\mathrm{mgu}(A,B)}{\sigma(A)\vee\sigma(C)}$$

CNF Transformation

$$\frac{C(A\vee B)^{\mathrm{T}}}{C\vee A^{\mathrm{T}}\vee B^{\mathrm{T}}}$$

$$\frac{C(A\vee B)^{\mathrm{F}}}{C\vee A^{\mathrm{F}} \; ; \; C\vee B^{\mathrm{F}}}$$

$$\frac{C\neg A^{\mathrm{T}}}{C\vee A^{\mathrm{F}}}$$

$$\frac{C\neg A^{\mathrm{F}}}{C\vee A^{\mathrm{T}}}$$

$$\frac{C\vee(A\Longrightarrow B)^{\mathrm{T}}}{C\vee A^{\mathrm{F}}\vee B^{\mathrm{T}}}$$

$$\frac{C\vee(A\Longrightarrow B)^{\mathrm{F}}}{C\vee A^{\mathrm{T}} \; ; \; C\vee B^{\mathrm{F}}}$$

$$\frac{C\vee(A\wedge B)^{\mathrm{T}}}{C\vee A^{\mathrm{T}} \; ; \; C\vee B^{\mathrm{T}}}$$

$$\frac{C\vee(A\wedge B)^{\mathrm{F}}}{C\vee A^{\mathrm{F}}\vee B^{\mathrm{F}}}$$

$$\frac{(\forall X.\ A)^{\mathrm{T}}\vee C \qquad Z\notin(\mathrm{free}(A)\cup\mathrm{free}(C))}{([Z/X]A)^{\mathrm{T}}\vee C}$$

$$\frac{(\forall X.\ A)^{\mathrm{F}}\vee C \qquad \{X_1,...,X_k\}=\mathrm{free}(\forall X.\ A)}{([f(X_1,...,X_k)/X]A)^{\mathrm{F}}\vee C} \quad f\in\Sigma_k^{\mathrm{sk}}\ \mathrm{new}$$

$$\frac{(\exists X.\ A)^{\mathrm{T}}\vee C \qquad \{X_1,...,X_k\}=\mathrm{free}(\exists X.\ A)}{([f(X_1,...,X_k)/X]A)^{\mathrm{T}}\vee C} \quad f\in\Sigma_k^{\mathrm{sk}}\ \mathrm{new}$$

$$\frac{(\exists X.\ A)^{\mathrm{F}}\vee C \qquad Z\notin(\mathrm{free}(A)\cup\mathrm{free}(C))}{([Z/X]A)^{\mathrm{F}}\vee C}$$

4 $\mathcal{ALC}$-Tableau

$$\frac{\begin{array}{c}x:c\\x:\overline{c}\end{array}}{\bot}$$

$$\frac{x:\varphi\sqcap\psi}{\begin{array}{c}x:\varphi\\x:\psi\end{array}}$$

$$\frac{x:\varphi\sqcup\psi}{x:\varphi \quad | \quad x:\psi}$$

$$\frac{\begin{array}{c}x:\forall\mathrm{R}.\ \varphi\\x\mathrm{R}y\end{array}}{y:\varphi}$$

$$\frac{x:\exists\mathrm{R}.\ \varphi}{\begin{array}{c}x\mathrm{R}y\\y:\varphi\end{array}}$$